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| March 25, 2015 | Matt Landreman |

# Effect on fluxes of the poloidally varying electrostatic potential

In these notes, we consider the following questions. Suppose neoclassical code #1 computes the particle and heat fluxes, totally ignoring the poloidal variation of the electrostatic potential. Then suppose neoclassical code #2 solves the drift-kinetic equation for the same gradients but now including the parallel acceleration force from the poloidally varying potential in the kinetic equation. In code #2, several variants of the radial fluxes can be computed, as there is now both a radial magnetic and ExB drift, and the radial transport of the poloidally varying electrostatic energy becomes important. How exactly are the fluxes in code #1 and code #2 related, and what are the “correct” definitions of the fluxes to use in code #2? In the end we will find that code #2 ends up getting the same answer for both the particle and energy flux as code #1.

All of the discussion here is relevant to both tokamaks and stellarators. For simplicity here we will neglect the  term which is important in stellarators but not (usually) in tokamaks. Some of the analysis may need to be re-examined when  is retained.

Suppose we have a solution  to the linear drift-kinetic equation in which there is no parallel variation of the electrostatic potential, as in DKES:

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Here,



is a stationary Maxwellian,  denotes species, and  is the magnetic drift (not including any  drift). We neglect the radial electric field in for simplicity. The solution  gives rise to a certain particle flux



and an energy flux

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(The subscript  on the fluxes - indicates these fluxes are associated with the radial *magnetic* drift.)

Now consider a more sophisticated drift-kinetic equation, in which parallel variation of the electrostatic potential  is retained. We denote the solution to this form of the kinetic equation by :

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The solution to can be written in terms of the solution to :

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If we evaluate the radial particle and energy fluxes associated with  caused by just the magnetic drifts (ignoring the radial  drift,) we do not get the same fluxes we got before:



and



In other words, including the blue  term in causes the radial fluxes to change by the red terms in -. Using

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we can evaluate the  integrals in the red terms of -, giving



and

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Using the MHD equilibrium relation , along with

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then - can be integrated by parts to find



and

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When  varies on a flux surface, there will also be a radial  drift. The particle and heat fluxes associated with this drift are, considering just  as opposed to ,



and

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Combining -, we find



and

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Eq shows that ignoring  in the kinetic equation does not change the particle flux: we get the same particle flux if we do include  and then account for the extra radial flux from the radial  drift. However, shows that the flux of kinetic energy *does* change when  is included in the kinetic equation, and this change cannot be compensated by accounting for the radial flux of kinetic energy from the radial  drift.

However, as Per points out in [1], the flux of *kinetic* energy is less physically relevant than the flux of total (kinetic + potential) energy. This flux of total energy is



where  is the total distribution function. Writing  where  and , and noting



(when ), then formally the largest term in in the  expansion is

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However vanishes exactly. The next-order terms in are



Some of these terms are quantities we have already considered:

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Also, the first right-hand-side term in is the same integral in red in , which we evaluated in going from to to . This term is one we neglected in computing the flux of kinetic energy , and it represents radial transport of the poloidally varying electrostatic energy. Thus, we find

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As one can see from -, can be written in terms of the particle and heat fluxes that would be found in the absence of :

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Thus, to leading order in , the physically meaningful flux should come out to be the same whether or not  is included in the computation.

[1] Helander, PPCF **37**, 57 (1995).